Appl Nanosci (2013) 3:281–290
DOI 10.1007/s13204-012-0129-1

ORIGINAL ARTICLE

Using serial radiation processing and microwave annealing decreases depth of *p–n* junction in a semiconductor heterostructure

E. L. Pankratov

Received: 29 November 2011 / Accepted: 25 April 2012 / Published online: 29 May 2012

Abstract It has recently been shown that manufacturing of diffusive-junction rectifiers and implanted-junction rectifiers in a semiconductor heterostructure after appropriate choosing of parameters of the structure and optimization of annealing time leads to increase of the sharpness of *p–n* junction and at one time to increase the homogeneity of dopant distribution in doped area. Formation of inhomogeneity of temperature in the heterostructure by laser or microwave annealing gives us possibility to increase the both effects at one time. It has recently been shown by experiments that predoping radiation processing of materials leads to changing of dopant diffusion in comparison with nonprocessed one. In this paper, we consider the possibility to use serial radiation processing of materials of heterostructure before doping and microwave annealing of radiation defects after doping to increase the sharpness *p–n* junctions and at one time to increase the homogeneity of dopant distribution in doped area in the heterostructure.

Keywords Increasing of sharpness of *p–n* junctions · Semiconductor heterostructure · Radiation processing · Microwave processing

Introduction

In the present time, intensive increasing of density of elements (such as *p–n* junctions and their systems) of integrated circuits (IC), performance and reliability of the elements occur (Grebene 1983; Gotra 1991; Lachin et al. 2001). At the same time, decreasing of depth of the elements of IC also occurs. One way to increase the performance is to determine new materials with higher speed of charge carrier transport (Grebene 1983; Gotra 1991; Lachin et al. 2001). Another way to increase the performance and also performance, reliability and density of elements of IC and to decrease depth of the elements is to elaborate new technological approaches or optimized existing technological approaches (Grebene 1983; Gotra 1991; Lachin et al. 2001; Sisianu et al. 2002; Pokotilo et al. 2006; Ahlgren et al. 1997; Volokobinskaya et al. 2001). For example, to increase of the sharpness of *p–n* junctions inhomogeneity of temperature distribution could be used, which could be obtained by laser or microwave annealing. To decrease the sharpness defects of doped material could be also used. In this paper we consider alternative approach to increase the sharpness of *p–n* junctions. To framework the approach we consider a semiconductor heterostructure (SH), which consists of a substrate (*S*) and two epitaxial layers (EL) (see Fig. 1). Let us denote the nearest EL to the *S* as EL_1. Let us also denote another EL as EL_2. We assume that types of conductivity of EL_2 and *S* are known and equal to each other (*p* or *n*). A dopant has been implanted in EL_1 to produce another type of conductivity in the layer (*n* or *p*). Further annealing of radiation defects has been done. It is practicable to choose thickness of EL_1 and energy of ions not independently from each other. After finishing of annealing the dopant should achieve both interfaces between layers of *H*. In this case near the interfaces two *p–n* junctions have been produced. If materials of the *H* are appropriately chosen, the interfaces give us possibility to increase at one time sharpness of the *p–n* junctions and homogeneity of dopant distribution in doped area in comparison with *p–n* junctions in homogenous

E. L. Pankratov (✉)
Nizhny Novgorod State University of Architecture and Civil Engineering, 65 Il'insky street, Nizhny Novgorod 603950, Russia
e-mail: elp2004@mail.ru

E. L. Pankratov
Nizhny Novgorod Institute of Commerce, 27 Lenin Avenue, Nizhny Novgorod 603950, Russia

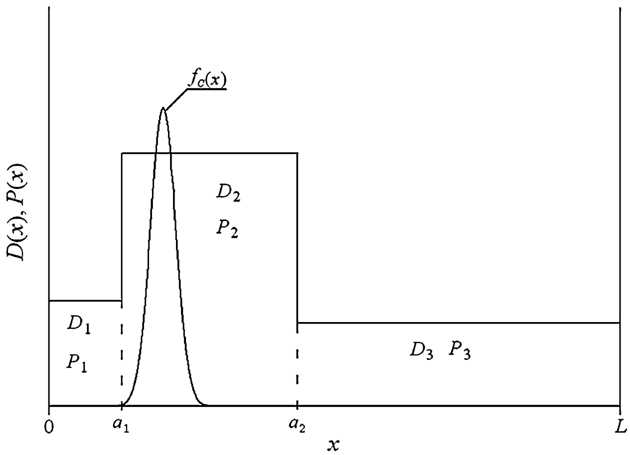

Fig. 1 Heterostructure with two epitaxial layers ($x \in [0, a_1]$ and $x \in [a_1, a_2]$) and substrate ($x \in [a_2, L]$). The figure also illustrates initial (before starting of annealing) distribution of dopant

materials (Pankratov 2008a, b). If the dopant does not achieve the interfaces during annealing, it is practicable to anneal the dopant additionally to shift the *p–n* junctions to the interfaces. It is known (see, for example, (Sobolev 2010; Kozlivsky 2003; Zorin et al. 1975) that dopant diffusion in materials after radiation processing differs from dopant diffusion in materials without radiation processing. The radiation processing leads to acceleration of the diffusion. Therefore, it is practicable to make radiation processing of the epitaxial layers before implantation of dopant into the EL_1. The processing leads to increasing of difference between diffusion coefficients of dopant in the epitaxial layers and the *S* of the SH. In this case, one can obtain increasing of the sharpness of the *p–n* junctions and increasing of dopant distribution in doped area in comparison with nonprocessed SH. On the other hand, the radiation processing leads to acceleration of dopant diffusion during annealing of radiation defects from the EL_1 to the EL_2. In this situation, we obtain more graded *p–n* junction between the epitaxial layers in comparison with *p–n* junction between the EL_1 and the *S*. To decrease the diffusion, it is practical to use microwave annealing of radiation defects in the EL_2 to decrease quantity of the radiation defects in the epitaxial layer. In this case, frequency of electromagnetic field should be appropriately chosen. After the microwave annealing, it could be used again to anneal of the radiation defects in the both epitaxial layer. The second annealing leads to increasing of the difference between dopant diffusion coefficients in the EL_1 and in the *S*. But in this situation we cannot obtain the additional difference between dopant diffusion coefficients in the epitaxial layers. Therefore, we obtain the *p–n* junction between the EL_1 and the *S* with higher sharpness in comparison with *p–n* junction between the EL_1 and the EL_2. The aims of the present paper were (a) optimization of continuance of additional annealing of dopant to shift the *p–n* junctions to the interfaces between layers of SH and (b) elaboration of mathematical approach to modeling of spatiotemporal distributions of temperature and dopant and radiation defect concentrations.

Method of solution

To solve our aim let us determine spatiotemporal distribution of dopant concentration. The distribution we determine by solving the second Fick's law (Grebene 1983: Gotra 1991; Lachin et al. 2001)

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_C \frac{\partial C(x,t)}{\partial x}\right] - k_C(x,T)\, C(x,t)\, V(x,t) \quad (1)$$

with boundary and initial conditions

$$\frac{\partial C(0,t)}{\partial x} = \frac{\partial C(L,t)}{\partial x} = 0, C(x,0) = f_C(x). \quad (2)$$

Here $C(x, t)$ is the spatiotemporal distribution of dopant concentration, D_C is the dopant diffusion coefficient, $V(x, t)$ is the spatiotemporal distribution of concentration of vacancies and $k_C(x, T)$ is the parameter of interaction between radiation vacancies and atoms of dopant. The value of dopant diffusion coefficient depends on properties of materials of layers in SH, on rate of heating and cooling of SH and on spatiotemporal distribution of dopant concentration. Concentrational dependence of diffusion coefficient could be approximated by the following functions (Gotra 1991; Kozlivsky 2003):

$$D_C = D_L(x,T)\left[1 + \xi \frac{C^{\gamma}(x,t)}{P^{\gamma}(x,T)}\right]\left[1 + \varsigma \frac{V(x,t)}{V^*} + \zeta \frac{I(x,t)}{I^*}\right]. \quad (3)$$

Here, $P(x, T)$ is the limit of solubility of dopant in SH; $D_L(x, T)$ is the diffusion coefficient for small quantity of dopant; parameter γ depends on properties of materials of SH and could be integer usually in the interval $\gamma \in [1, 3]$ (Gotra 1991); $I(x, t)$ is the spatiotemporal distribution of concentration of interstitials; and I^* and V^* are the equilibrium distributions of the same defects. Spatiotemporal distributions of point radiation defects (both, vacancies and interstitials) we determine by solving the following system of equations (Zorin et al. 1975; Ryssel and Ruge 1978; Fahey et al. 1989):

$$\begin{cases} \dfrac{\partial I(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_I(x,T)\dfrac{\partial I(x,t)}{\partial x}\right] - k_{I,V}(x,T) I(x,t) V(x,t) \\ \dfrac{\partial V(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_V(x,T)\dfrac{\partial V(x,t)}{\partial x}\right] - k_{I,V}(x,T) I(x,t) V(x,t) \end{cases} \quad (4)$$

with boundary and initial conditions

$$\frac{\partial\rho(0,t)}{\partial x}=\frac{\partial\rho(L,t)}{\partial x}=0,\rho(x,0)=f_{\rho}(x). \tag{5}$$

Here $\rho = I$, V; $D_{\rho}(x, T)$ are diffusion coefficients of interstitials and vacancies, and $k_{I,V}(x, T)$ is the parameter of recombination of point radiation defects. The initial distribution of radiation defects contains in itself two specific initial distributions. One of them has been induced during radiation processing of materials of SH. Another one has been induced during ion implantation.

Spatiotemporal distribution of temperature during annealing has been determined by the second low of Fourier in the following form (Shalimova 1985):

$$\begin{aligned}v(T)\frac{\partial T(x,t)}{\partial t}&=\frac{\partial}{\partial x}\left[\lambda\frac{\partial T(x,t)}{\partial x}\right]+p(x,t)\\&=p(x,t)-\frac{\partial J_T(x,t)}{\partial x},\end{aligned} \tag{6}$$

with boundary and initial conditions

$$J_T(0,\mathrm{t})=0,J_T(L,t)=0,T(x,0)=f_T(x). \tag{7}$$

Here $T(x, t)$ is the spatiotemporal distribution of temperature; $v(T) = v_{ass}[1 - \eta\ \exp(-T(x, t)/T_d)]$ is the heat capacitance of SH (in the most interest interval of temperature, when temperature $T(x, t)$ is approximately equal or larger, than Debye temperature T_d, it could be assumed $v(T) \approx v_{ass}$ (Shalimova 1985); λ is the heat conduction coefficient, which value depends on properties of materials of SH and temperature (dependence of heat conduction coefficient on temperature in the most interest interval of values of temperature could be approximated by the following function: $\lambda(x, T) = \lambda_{ass}(x)[1 + \mu(T_d/T(x, t))^{\varphi}]$; $p(x, t)$ is volumetric density of heating power of SH; $\alpha(x, T) = \lambda(x, T)/v(T)$.

First of all let us determine spatiotemporal distribution of temperature. To solve the Eq. (6) in pursuance (Pankratov 2005; Pankratov and Spagnolo 2005; Pankratov and Bulaeva 2012) we transform the approximation of the heat conduction coefficient $\alpha_{ass}(x) = \alpha_{0ass}[1 + \varepsilon_T g_T(x)]$, where $\lambda_{ass}(x)$ is the average value of the function $\lambda_{ass}(x)$. Further we determine solution of the Eq. (6) as a following power series:

$$T(x,t)=\sum_{i=1}^{\infty}\varepsilon_T^i\sum_{j=1}^{\infty}\mu^jT_{ij}(x,t). \tag{8}$$

Substitution of the series Eq. (7) in Eq. (6) gives us possibility to obtain the system of zero-order approximation of temperature $T_{00}(x, t)$ and corrections to the approximation $T_{ij}(x, t)$ ($i \geq 1$, $j \geq 1$) in the following form:

$$\begin{aligned}
&\frac{\partial T_{00}(x,t)}{\partial t}=\alpha_{0ass}\frac{\partial^2T_{00}(x,t)}{\partial x^2}+\frac{p(x,t)}{v_{ass}}\\
&\frac{\partial T_{i0}(x,t)}{\partial t}=\alpha_{0ass}\frac{\partial^2T_{i0}(x,t)}{\partial x^2}+\alpha_{0ass}\frac{\partial}{\partial x}\left[g_T(x)\frac{\partial T_{i-10}(x,t)}{\partial x}\right]\\
&\quad i\geq1\\
&\frac{\partial T_{01}(x,t)}{\partial t}=\alpha_{0ass}\frac{\partial^2T_{01}(x,t)}{\partial x^2}+\frac{\alpha_{0ass}T_d^{\phi}}{T_{00}^{\phi}(x,t)}\frac{\partial^2T_{00}(x,t)}{\partial x^2}\\
&\qquad-\frac{\phi\,\alpha_{0ass}T_d^{\phi}}{T_{00}^{\phi+1}(x,t)}\left[\frac{\partial T_{00}(x,t)}{\partial x}\right]^2\\
&\frac{\partial T_{02}(x,t)}{\partial t}=\alpha_{0ass}\frac{\partial^2T_{02}(x,t)}{\partial x^2}+\frac{\alpha_{0ass}T_d^{\phi}}{T_{00}^{\phi}(x,t)}\frac{\partial^2T_{01}(x,t)}{\partial x^2}\\
&\qquad-\frac{\phi\,\alpha_{0ass}T_d^{\phi}}{T_{00}^{\phi+1}(x,t)}\frac{\partial T_{00}(x,t)}{\partial x}\frac{\partial T_{01}(x,t)}{\partial x}\\
&\frac{\partial T_{11}(x,t)}{\partial t}=\alpha_{0ass}\frac{\partial^2T_{11}(x,t)}{\partial x^2}\\
&\qquad+\alpha_{0ass}\frac{T_{01}(x,t)}{T_{00}(x,t)}\frac{\partial}{\partial x}\left[g_T(x)\frac{\partial T_{00}(x,t)}{\partial x}\right]\\
&\qquad+\alpha_{0ass}\frac{\partial^2T_{01}(x,t)}{\partial x^2}\frac{\partial}{\partial x}\left[g_T(x)\frac{\partial T_{01}(x,t)}{\partial x}\right]\\
&\qquad+\alpha_{0ass}\frac{T_d^{\phi}T_{10}(x,t)}{T_{00}^{\phi+1}(x,t)}\frac{\partial^2T_{00}(x,t)}{\partial x^2}\\
&\qquad+\frac{\alpha_{0ass}T_d^{\phi}}{T_{00}^{\phi}(x,t)}\frac{\partial^2T_{10}(x,t)}{\partial x^2}\\
&\qquad+\frac{\alpha_{0ass}T_d^{\phi}}{T_{00}^{\phi}(x,t)}\frac{\partial}{\partial x}\left[g_T(x)\frac{\partial T_{00}(x,t)}{\partial x}\right]\\
&\qquad-\frac{\phi\alpha_{0ass}T_d^{\phi}}{T_{00}^{\phi+1}(x,t)}\frac{\partial T_{10}(x,t)}{\partial x}\frac{\partial T_{00}(x,t)}{\partial x}\\
&\qquad-g_T(x)\frac{\phi\alpha_{0ass}T_d^{\phi}}{T_{00}^{\phi+1}(x,t)}\left[\frac{\partial T_{00}(x,t)}{\partial x}\right]^2.
\end{aligned} \tag{9}$$

Substitution of the series Eq. (8) into the boundary and initial conditions Eq. (7) leads to the boundary and initial conditions for the all functions $T_{ij}(x, t)$

$$\begin{aligned}&T_{00}(x,0)=f_T(x),T_{ij}(x,0)=0;\frac{\partial T_{ij}(0,t)}{\partial x}=\frac{\partial T_{ij}(L,t)}{\partial x}=0,\\&\qquad i\geq1,j\geq1.\end{aligned} \tag{10}$$

Solution of the Eq. (9) with conditions Eq. (10) gives us the following result:

$$T_{00}(x,t) = \frac{1}{L}\int_0^L f_T(v)\,\mathrm{d}v + \frac{2}{L}\sum_{n=1}^{\infty} c_n(x)\,e_{nT}(t)\int_0^L c_n(v) f_T(v)\,\mathrm{d}v + \frac{1}{L}\int_0^L \frac{p(v,t)}{v_{\mathrm{ass}}}\,\mathrm{d}v + \frac{2}{L}\sum_{n=1}^{\infty} c_n(x)\,e_{nT}(t)\int_0^t e_{nT}(-\tau) \times \int_0^L c_n(v)\frac{p(v,\tau)}{v_{\mathrm{ass}}}\,\mathrm{d}v\,\mathrm{d}\tau,$$

where $c_n(x) = \cos(\pi\, n\, x/L)$, $e_{nT}(t) = \exp(-\pi^2 n^2 \alpha_{0\mathrm{ass}} t/L^2)$;

$$T_{i0}(x,t) = \alpha_{0\mathrm{ass}}\frac{2\pi}{L^2}\sum_{n=1}^{\infty} n\, c_n(x)\,e_{nT}(t)\int_0^t e_{nT}(-\tau) \times \int_0^L s_n(v)\,g_T(v)\frac{\partial T_{i-10}(v,\tau)}{\partial v}\,\mathrm{d}v\,\mathrm{d}\tau, \quad i \geq 1;$$

where $F_{nT} = \int_0^L f_T(v) c_n(v)\,\mathrm{d}v$, $s_n(x) = \sin(\pi n x/L)$;

$$T_{01}(x,t) = \alpha_{0\mathrm{ass}} T_{\mathrm{d}}^{\phi}\frac{2\pi}{L^2}\sum_{n=1}^{\infty} n\, c_n(x) e_{nT}(t) \times \int_0^t e_{nT}(-\tau)\int_0^L \frac{s_n(v)}{T_{00}^{\phi}(v,\tau)}\frac{\partial^2 T_{00}(v,\tau)}{\partial v^2}\,\mathrm{d}v\,\mathrm{d}\tau - \phi\alpha_{0\mathrm{ass}} T_{\mathrm{d}}^{\phi}\frac{2}{L}\sum_{n=1}^{\infty} c_n(x) e_{nT}(t)\int_0^t e_{nT}(-\tau) \times \int_0^L \frac{c_n(v)}{T_{00}^{\phi+1}(v,\tau)}\left[\frac{\partial T_{00}(v,\tau)}{\partial v}\right]^2 \mathrm{d}v\,\mathrm{d}\tau;$$

$$T_{02}(x,t) = \alpha_{0\mathrm{ass}} T_{\mathrm{d}}^{\phi}\frac{2\pi}{L^2}\sum_{n=1}^{\infty} n\, c_n(x)\,e_{nT}(t)\int_0^t e_{nT}(-\tau) \times \int_0^L \frac{s_n(v)}{T_{00}^{\phi}(v,\tau)}\frac{\partial^2 T_{01}(v,\tau)}{\partial v^2}\,\mathrm{d}v\,\mathrm{d}\,\tau - \phi\,\alpha_{0\mathrm{ass}} T_{\mathrm{d}}^{\phi}\frac{2}{L^2}\sum_{n=1}^{\infty} c_n(x)\,e_{nT}(t)\int_0^t e_{nT}(-\tau) \times \int_0^L \frac{c_n(v)}{T_{00}^{\phi+1}(v,\tau)}\frac{\partial T_{00}(v,\tau)}{\partial v}\frac{\partial T_{01}(v,\tau)}{\partial v}\,\mathrm{d}v\,\mathrm{d}\tau;$$

$$T_{11}(x,t) = \alpha_{0\mathrm{ass}}\frac{2}{L}\sum_{n=1}^{\infty} c_n(x)\,e_{nT}(t)\int_0^t e_{nT}(-\tau) \times \int_0^L c_n(v)\frac{T_{01}(v,\tau)}{T_{00}(v,\tau)}\frac{\partial}{\partial v}\left[g_T(v)\frac{\partial T_{00}(v,\tau)}{\partial v}\right]\mathrm{d}v\,\mathrm{d}\tau + \alpha_{0\mathrm{ass}}\frac{2}{L}\sum_{n=1}^{\infty} c_n(x)\,e_{nT}(t)\int_0^t e_{nT}(-\tau) \times \int_0^L c_n(v)\frac{\partial}{\partial v}\left[g_T(v)\frac{\partial T_{01}(v,\tau)}{\partial v}\right]\mathrm{d}v\,\mathrm{d}\tau + \alpha_{0\mathrm{ass}} T_{\mathrm{d}}^{\phi}\frac{2}{L} \times \sum_{n=1}^{\infty} n\, c_n(x) e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^L \frac{c_n(v)}{T_{00}^{\phi}(v,\tau)}\frac{\partial^2 T_{10}(v,\tau)}{\partial v^2}\,\mathrm{d}v\,\mathrm{d}\tau + \alpha_{0\mathrm{ass}} T_{\mathrm{d}}^{\phi}\frac{2}{L}\sum_{n=1}^{\infty} n\, c_n(x)\,e_{nT}(t) \times \int_0^t e_{nT}(-\tau)\int_0^L \frac{c_n(v)\,T_{10}(v,\tau)}{T_{00}^{\phi+1}(v,\tau)}\frac{\partial^2 T_{10}(v,\tau)}{\partial v^2}\,\mathrm{d}v\,\mathrm{d}\tau + \alpha_{0\mathrm{ass}} T_{\mathrm{d}}^{\phi}\frac{2}{L}\sum_{n=1}^{\infty} n\, c_n(x)\,e_{nT}(t)\int_0^t e_{nT}(-\tau) \times \int_0^L \frac{c_n(v)}{T_{00}^{\phi}(v,\tau)}\frac{\partial}{\partial v}\left[g_T(v)\frac{\partial T_{00}(v,\tau)}{\partial v}\right]\mathrm{d}v\,\mathrm{d}\tau - 2\,\alpha_{0\mathrm{ass}}\frac{T_{\mathrm{d}}^{\phi}}{L}\sum_{n=1}^{\infty} n\, c_n(x)\,e_{nT}(t)\int_0^t e_{nT}(-\tau) \times \int_0^L \frac{c_n(v)}{T_{00}^{\phi+1}(v,\tau)}\frac{\partial T_{10}(v,\tau)}{\partial v}\frac{\partial T_{00}(v,\tau)}{\partial v}\,\mathrm{d}v\,\mathrm{d}\tau - \alpha_{0\mathrm{ass}} T_{\mathrm{d}}^{\phi}\,\phi\,\frac{2}{L}\sum_{n=1}^{\infty} n\, c_n(x)\,e_{nT}(t)\int_0^t e_{nT}(-\tau) \times \int_0^L \frac{c_n(v) g_T(v)}{T_{00}^{\phi+1}(v,\tau)}\left[\frac{\partial T_{00}(v,\tau)}{\partial v}\right]^2 \mathrm{d}v\,\mathrm{d}\tau.$$

To analyze spatiotemporal distribution of temperature qualitatively and to obtain some quantitative results the second-order approximations of temperature are good enough (see, for example, Shalimova 1985; Pankratov 2005). Analytical results give us possibility to obtain and to illustrate demonstrably main physical dependencies. To obtain the results with higher exactness numerical approaches have been used. Further to determine spatiotemporal distribution of concentration of radiation defects, we transform approximations of diffusion coefficients of defects and parameter of recombination to the following form: $D_\rho(x, T) = D_{0\rho}[1 + \varepsilon_\rho g_\rho(x, T)]$ and $k_{I,V}(x, T) = k_{0I,V}[1 + \zeta h(x, T)]$. Here $D_{0\rho}$ and $k_{0I,V}$ are average values of the appropriate values, $0 \le \varepsilon_\rho < 1$, $0 \le \zeta < 1$, $|g_\rho(x, T)| \le 1$, $|h(x, T)| \le 1$. Let us introduce the following dimensionless variables: $\vartheta = t\sqrt{D_{0I}D_{0V}}/L^2$, $\omega = k_{0I,V}L^2\sqrt{I^*V^*}/\sqrt{D_{0I}D_{0V}}$, $\chi = x/L$. After the introduction the Eq. (4) takes the form

$$\begin{cases} \dfrac{\partial \tilde{I}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial}{\partial \chi}\left\{[1+\varepsilon_I g_I(\chi,T)]\dfrac{\partial \tilde{I}(\chi,\vartheta)}{\partial \chi}\right\} \\ \qquad -\omega[1+\varsigma h(\chi,T)]\sqrt{\dfrac{V^*}{I^*}}\tilde{I}(\chi,\vartheta)\tilde{V}(\chi,\vartheta) \\ \dfrac{\partial \tilde{V}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}}\dfrac{\partial}{\partial \chi}\left\{[1+\varepsilon_V g_V(\chi,T)]\dfrac{\partial \tilde{V}(\chi,\vartheta)}{\partial \chi}\right\} \\ \qquad -\omega[1+\varsigma h(\chi,T)]\sqrt{\dfrac{I^*}{V^*}}\tilde{I}(\chi,\vartheta)\tilde{V}(\chi,\vartheta) \end{cases} \tag{11}$$

Let us determine solution of Eq. (11) as the following power series:

$$\tilde{\rho}(\chi,\vartheta) = \sum_{i=0}^{\infty}\varepsilon_{\tilde{\rho}}^{i}\sum_{j=0}^{\infty}\omega^{j}\sum_{k=0}^{\infty}\varsigma^{k}\tilde{\rho}_{ijk}(\chi,\vartheta). \tag{12}$$

Substitution of the series in the system of equations Eq. (11) gives us possibility to obtain equations for zeroth-order approximations of concentrations of defects $\tilde{\rho}_{000}(\chi,\vartheta)$ and corrections to the approximations $\tilde{\rho}_{ijk}(\chi,\vartheta)$ ($i \ge 1$, $j \ge 1$, $k \ge 1$) in the following form:

$$\begin{cases} \dfrac{\partial \tilde{I}_{000}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial^2 \tilde{I}_{000}(\chi,\vartheta)}{\partial \chi^2} \\ \dfrac{\partial \tilde{V}_{000}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}}\dfrac{\partial^2 \tilde{V}_{000}(\chi,\vartheta)}{\partial \chi^2} \end{cases};$$

$$\begin{cases} \dfrac{\partial \tilde{I}_{i00}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial^2 \tilde{I}_{i00}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad +\sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial}{\partial \chi}\left[g_I(\chi,T)\dfrac{\partial \tilde{I}_{i-100}(\chi,\vartheta)}{\partial \chi}\right] \\ \dfrac{\partial \tilde{V}_{i00}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}}\dfrac{\partial^2 \tilde{V}_{i00}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad +\sqrt{\dfrac{D_{0V}}{D_{0I}}}\dfrac{\partial}{\partial \chi}\left[g_V(\chi,T)\dfrac{\partial \tilde{V}_{i-100}(\chi,\vartheta)}{\partial \chi}\right] \end{cases},$$

$i \ge 1$;

$$\begin{cases} \dfrac{\partial \tilde{I}_{010}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial^2 \tilde{I}_{010}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad -\sqrt{\dfrac{V^*}{I^*}}\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \\ \dfrac{\partial \tilde{V}_{010}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}}\dfrac{\partial^2 \tilde{V}_{010}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad -\sqrt{\dfrac{V^*}{I^*}}\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \end{cases};$$

$$\begin{cases} \dfrac{\partial \tilde{I}_{020}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial^2 \tilde{I}_{020}(\chi,\vartheta)}{\partial \chi^2} - \sqrt{\dfrac{V^*}{I^*}}\tilde{I}_{010}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \\ \qquad -\sqrt{\dfrac{V^*}{I^*}}\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{010}(\chi,\vartheta) \\ \dfrac{\partial \tilde{V}_{020}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial^2 \tilde{V}_{020}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad -\sqrt{\dfrac{V^*}{I^*}}\tilde{I}_{010}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \\ \qquad -\sqrt{\dfrac{V^*}{I^*}}\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{010}(\chi,\vartheta) \end{cases};$$

$$\begin{cases} \dfrac{\partial \tilde{I}_{00k}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial^2 \tilde{I}_{00k}(\chi,\vartheta)}{\partial \chi^2} \\ \dfrac{\partial \tilde{V}_{00k}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}}\dfrac{\partial^2 \tilde{V}_{00k}(\chi,\vartheta)}{\partial \chi^2} \end{cases}; \tag{13}$$

$$\begin{cases} \dfrac{\partial \tilde{I}_{110}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial^2 \tilde{I}_{110}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad +\sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial}{\partial \chi}\left[g_I(\chi,T)\dfrac{\partial \tilde{I}_{010}(\chi,\vartheta)}{\partial \chi}\right] \\ \qquad -\sqrt{\dfrac{V^*}{I^*}}\tilde{I}_{100}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \\ \qquad -\sqrt{\dfrac{V^*}{I^*}}\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{100}(\chi,\vartheta) \\ \dfrac{\partial \tilde{V}_{110}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}}\dfrac{\partial^2 \tilde{V}_{110}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad +\sqrt{\dfrac{D_{0V}}{D_{0I}}}\dfrac{\partial}{\partial \chi}\left[g_V(\chi,T)\dfrac{\partial \tilde{V}_{010}(\chi,\vartheta)}{\partial \chi}\right] \\ \qquad -\sqrt{\dfrac{V^*}{I^*}}\tilde{I}_{100}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \\ \qquad -\sqrt{\dfrac{V^*}{I^*}}\tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{100}(\chi,\vartheta) \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{101}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{101}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad + \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \chi}\left[g_I(\chi,T)\frac{\partial \tilde{I}_{001}(\chi,\vartheta)}{\partial \chi}\right] \\ \frac{\partial \tilde{V}_{101}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{101}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad + \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \chi}\left[g_V(\chi,T)\frac{\partial \tilde{V}_{001}(\chi,\vartheta)}{\partial \chi}\right] \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{011}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{011}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad - \sqrt{\frac{V^*}{I^*}} \tilde{I}_{001}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \\ \qquad - \sqrt{\frac{V^*}{I^*}} \tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{001}(\chi,\vartheta) \\ \qquad - h(\chi,T)\sqrt{\frac{V^*}{I^*}} \tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \\ \frac{\partial \tilde{V}_{011}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{011}(\chi,\vartheta)}{\partial \chi^2} \\ \qquad - \sqrt{\frac{V^*}{I^*}} \tilde{I}_{001}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta) \\ \qquad - \sqrt{\frac{V^*}{I^*}} \tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{001}(\chi,\vartheta) \\ \qquad - h(\chi,T)\sqrt{\frac{V^*}{I^*}} \tilde{I}_{000}(\chi,\vartheta)\tilde{V}_{000}(\chi,\vartheta). \end{cases}$$

Substitution of the series (12) into appropriate boundary and initial conditions gives us possibility to obtain boundary and initial conditions for the functions required $\tilde{\rho}_{ijk}(\chi,\vartheta)$ in the following form:

$$\left.\frac{\partial \tilde{\rho}_{ijk}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=0} = \left.\frac{\partial \tilde{\rho}_{ijk}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=1} = 0;\ \tilde{\rho}_{000}(\chi,0) = \frac{f_\rho(\chi)}{\rho^*};\ \tilde{\rho}_{ijk}(\chi,0) = 0, i \ge 1, j \ge 1, k \ge 1.$$

Solutions of Eq. (13) are

$$\tilde{\rho}_{000}(\chi,\vartheta) = \frac{F_{0\rho}}{\rho^* L} + \frac{2}{\rho^* L}\sum\nolimits_{n=1}^{\infty} F_{n\rho} c_n(\chi) e_{n\rho}(\vartheta),$$

where $F_{n\rho} = \int_0^1 f_\rho(\chi)\, c_n(\chi)\, d\chi c_n(\chi) = \cos(\pi n \chi), e_{nI}(\vartheta) = \exp\left(-\pi^2 n^2 \vartheta \sqrt{D_{0I}/D_{0V}}\right), e_{nV}(\vartheta) = \exp\left(-\pi^2 n^2 \vartheta \sqrt{D_{0V}/D_{0I}}\right)$;

$$\tilde{\rho}_{i00}(\chi,\vartheta) = -\frac{2\pi D_{0\rho}}{\rho^* L^2} \sum_{n=1}^{\infty} n c_n(\chi) e_{n\rho}(\vartheta) \int_0^{\vartheta} e_{n\rho}(-\tau) \times \int_0^1 s_n(v) g(v,T) \frac{\partial \tilde{\rho}_{i-100}(v,\tau)}{\partial v}\, dv\, d\tau, \quad i \ge 1;$$

$$\tilde{I}_{010}(\chi,\vartheta) = -\frac{2D_{0I}}{I^* L}\sqrt{\frac{V^*}{I^*}} \sum_{n=1}^{\infty} c_n(\chi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \times \int_0^1 c_n(v) \tilde{I}_{000}(v,\tau) \tilde{V}_{000}(v,\tau)\, dv\, d\tau;$$

$$\tilde{V}_{010}(\chi,\vartheta) = -\frac{2D_{0V}}{V^* L}\sqrt{\frac{I^*}{V^*}} \sum_{n=1}^{\infty} c_n(\chi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \times \int_0^1 c_n(v) \tilde{I}_{000}(v,\tau) \tilde{V}_{000}(v,\tau)\, dv\, d\tau;$$

$$\tilde{I}_{020}(\chi,\vartheta) = -\frac{2D_{0I}}{I^* L}\sqrt{\frac{V^*}{I^*}} \sum_{n=1}^{\infty} c_n(\chi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \times \int_0^1 c_n(v) \tilde{I}_{010}(v,\tau) \tilde{V}_{000}(v,\tau)\, dv\, d\tau - \frac{2D_{0I}}{I^* L}\sqrt{\frac{V^*}{I^*}} \sum_{n=1}^{\infty} c_n(\chi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \times \int_0^1 c_n(v) \tilde{I}_{000}(v,\tau) \tilde{V}_{010}(v,\tau)\, dv\, d\tau;$$

$$\tilde{V}_{020}(\chi,\vartheta) = -\frac{2D_{0V}}{V^* L}\sqrt{\frac{I^*}{V^*}} \sum_{n=1}^{\infty} c_n(\chi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \times \int_0^1 c_n(v) \tilde{I}_{010}(v,\tau) \tilde{V}_{000}(v,\tau)\, dv\, d\tau - \frac{2D_{0V}}{V^* L}\sqrt{\frac{I^*}{V^*}} \sum_{n=1}^{\infty} c_n(\chi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \times \int_0^1 c_n(v) \tilde{I}_{000}(v,\tau) \tilde{V}_{010}(v,\tau)\, dv\, d\tau;$$

$$\tilde{I}_{001}(\chi,\vartheta) = \tilde{I}_{002}(\chi,\vartheta) = \tilde{V}_{001}(\chi,\vartheta) = \tilde{V}_{002}(\chi,\vartheta) = 0;$$

$$\tilde{I}_{110}(\chi,\vartheta)=\frac{2\pi D_{0I}}{I^*L^2}\sqrt{\frac{V^*}{I^*}}\sum_{n=1}^{\infty}nc_n(\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\times\int_0^1 s_n(v)g_I(v,T)\frac{\partial\tilde{I}_{010}(v,\tau)}{\partial v}\,\mathrm{d}v\,\mathrm{d}\tau-\frac{2D_{0I}}{I^*L}\sqrt{\frac{V^*}{I^*}}\sum_{n=1}^{\infty}c_n(\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\times\int_0^1 c_n(v)\tilde{I}_{100}(v,\tau)\tilde{V}_{000}(v,\tau)\,\mathrm{d}v\,\mathrm{d}\tau-\frac{2D_{0I}}{I^*L}\sqrt{\frac{V^*}{I^*}}\times\sum_{n=1}^{\infty}c_n(\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\times\int_0^1 c_n(v)\tilde{I}_{000}(v,\tau)\tilde{V}_{100}(v,\tau)\,\mathrm{d}v\,\mathrm{d}\tau, \quad (14)$$

where $s_n(\chi) = \sin(\pi n\chi)$;

$$\tilde{V}_{110}(\chi,\vartheta)=\frac{2\pi D_{0V}}{V^*L^2}\sum_{n=1}^{\infty}nc_n(\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\times\int_0^1 s_n(v)g_V(v,T)\frac{\partial\tilde{V}_{010}(v,\tau)}{\partial v}\,\mathrm{d}v\,\mathrm{d}\tau\times\sqrt{\frac{I^*}{V^*}}-\frac{2D_{0V}}{V^*L}\sqrt{\frac{I^*}{V^*}}\sum_{n=1}^{\infty}c_n(\chi)e_{nV}(\vartheta)\times\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 c_n(v)\tilde{I}_{100}(v,\tau)\tilde{V}_{000}(v,\tau)\,\mathrm{d}v\,\mathrm{d}\tau-\frac{2D_{0V}}{V^*L}\sqrt{\frac{I^*}{V^*}}\sum_{n=1}^{\infty}c_n(\chi)e_{nV}(\vartheta)\times\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 c_n(v)\tilde{I}_{000}(v,\tau)\tilde{V}_{100}(v,\tau)\,\mathrm{d}v\,\mathrm{d}\tau;$$

$$\tilde{I}_{101}(\chi,\vartheta)=\frac{2\pi}{I^*L^2}\sum_{n=1}^{\infty}nc_n(\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\times\int_0^1 s_n(v)g_I(v,T)\frac{\partial\tilde{I}_{001}(v,\tau)}{\partial v}\,\mathrm{d}v\,\mathrm{d}\tau;$$

$$\tilde{V}_{101}(\chi,\vartheta)=\frac{2\pi}{V^*L^2}\sum_{n=1}^{\infty}nc_n(\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\times\int_0^1 s_n(v)g_V(v,T)\frac{\partial\tilde{V}_{001}(v,\tau)}{\partial v}\,\mathrm{d}v\,\mathrm{d}\tau;$$

$$\tilde{I}_{011}(\chi,\vartheta)=-\frac{2}{I^*L}\sqrt{\frac{V^*}{I^*}}\sum_{n=1}^{\infty}c_n(\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\times\int_0^1 c_n(v)\tilde{I}_{001}(v,\tau)\tilde{V}_{000}(v,\tau)\,\mathrm{d}v\,\mathrm{d}\tau-\frac{2}{I^*L}\sqrt{\frac{V^*}{I^*}}\sum_{n=1}^{\infty}c_n(\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\times\int_0^1 c_n(v)\tilde{I}_{000}(v,\tau)\tilde{V}_{001}(v,\tau)\,\mathrm{d}v\,\mathrm{d}\tau-\frac{2}{I^*L}\sqrt{\frac{V^*}{I^*}}\sum_{n=1}^{\infty}c_n(\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\times\int_0^1 c_n(v)h(v,T)\tilde{I}_{000}(v,\tau)\tilde{V}_{000}(v,\tau)\,\mathrm{d}v\,\mathrm{d}\tau$$

$$\tilde{V}_{011}(\chi,\vartheta)=-\frac{2}{V^*L}\sqrt{\frac{I^*}{V^*}}\sum_{n=1}^{\infty}c_n(\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\times\int_0^1 c_n(v)\tilde{I}_{001}(v,\tau)\tilde{V}_{000}(v,\tau)\,\mathrm{d}v\,\mathrm{d}\tau-\frac{2}{V^*L}\sqrt{\frac{I^*}{V^*}}\sum_{n=1}^{\infty}c_n(\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\times\int_0^1 c_n(v)\tilde{I}_{000}(v,\tau)\tilde{V}_{001}(v,\tau)\,\mathrm{d}v\,\mathrm{d}\tau-\frac{2}{V^*L}\sqrt{\frac{I^*}{V^*}}\sum_{n=1}^{\infty}c_n(\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\times\int_0^1 c_n(v)h(v,T)\tilde{I}_{000}(v,\tau)\tilde{V}_{000}(v,\tau)\,\mathrm{d}v\,\mathrm{d}\tau.$$

Analysis of redistribution of radiation defects has been done using the second-order approximation of spatiotemporal distribution of concentrations of the defects and by numerical approaches. Further, we solved Eq. (1). To obtain the solution let us transform dopant diffusion coefficient $D_L(x, T)$ and parameter of interaction $k_C(x, T)$ in the following forms: $D_L(x, T) = D_{0L}[1 + \varepsilon_L g_L(x, T)]$, $k_C(x, T) = k_{0C}[1 + \alpha q(x, T)]$, $0 \le \varepsilon_L < 1$, $0 \le \alpha < 1$, $|g_L(x,T)| \le 1$, $|q(x, T)| \le 1$ and to determine dopant concentration as the power series

$$C(x,t) = \sum_{i=0}^{\infty} \varepsilon_L^i \sum_{j=0}^{\infty} \xi^j \sum_{k=0}^{\infty} \alpha^k C_{ijk}(x,t). \tag{15}$$

Substitution of the series Eq. (15) in the equation Eq. (1) gives us possibility to obtain equations for zeroth-order approximations of concentration of dopant $C_{00}(x, t)$ and corrections to the approximations $C_{ij}(x, t)$ $(i \ge 1, j \ge 1)$ in the following form:

$$\frac{\partial C_{000}(x,t)}{\partial t} = D_{0C}\frac{\partial^2 C_{000}(x,t)}{\partial x^2};$$

$$\frac{\partial C_{i00}(x,t)}{\partial t} = D_{0C}\frac{\partial^2 C_{i00}(x,t)}{\partial x^2} + D_{0C}\frac{\partial}{\partial x}\left[g_L(x,T)\frac{\partial C_{i00}(x,t)}{\partial x}\right], \quad i \ge 1;$$

$$\frac{\partial C_{010}(x,t)}{\partial t} = D_{0C}\frac{\partial^2 C_{010}(x,t)}{\partial x^2} + D_{0C}\frac{\partial}{\partial x}\left[\frac{C_{000}^{\gamma}(x,t)}{P^{\gamma}(x,T)}\frac{\partial C_{000}(x,t)}{\partial x}\right];$$

$$\frac{\partial C_{020}(x,t)}{\partial t} = D_{0C}\frac{\partial^2 C_{020}(x,t)}{\partial x^2} + D_{0C}\frac{\partial}{\partial x}\left[C_{010}(x,t)\frac{C_{000}^{\gamma-1}(x,t)}{P^{\gamma}(x,T)}\frac{\partial C_{000}(x,t)}{\partial x}\right] + D_{0C}\frac{\partial}{\partial x}\left[\frac{C_{000}^{\gamma}(x,t)}{P^{\gamma}(x,T)}\frac{\partial C_{010}(x,t)}{\partial x}\right];$$

$$\begin{aligned}\frac{\partial C_{110}(x,t)}{\partial t} = {} & D_{0C}\frac{\partial^2 C_{110}(x,t)}{\partial x^2} + D_{0C}\frac{\partial}{\partial x}\left[g_L(x,T)\frac{\partial C_{010}(x,t)}{\partial x}\right] + D_{0C}\frac{\partial}{\partial x}\left[\frac{C_{000}^{\gamma}(x,t)}{P^{\gamma}(x,T)}\frac{\partial C_{100}(x,t)}{\partial x}\right] \\ & + D_{0C}\frac{\partial}{\partial x}\left[C_{10}(x,t)\frac{C_{000}^{\gamma-1}(x,t)}{P^{\gamma}(x,T)}\frac{\partial C_{000}(x,t)}{\partial x}\right] + D_{0C}\frac{\partial}{\partial x}\left[g_L(x,T)\frac{C_{000}^{\gamma}(x,t)}{P^{\gamma}(x,T)}\frac{\partial C_{000}(x,t)}{\partial x}\right];\end{aligned}$$

$$\frac{\partial C_{001}(x,t)}{\partial t} = -q_C(x,T)I^* V^* C_{001}(x,t)V(x,t);$$

$$\frac{\partial C_{002}(x,t)}{\partial t} = -q_C(x,T)I^* V^* C_{002}(x,t)V(x,t); \tag{16}$$

$\ldots \quad \ldots \quad \ldots$

Substitution of the series (15) into appropriate boundary and initial conditions gives us possibility to obtain boundary and initial conditions for the functions $C_{ij}(x,t)$ in the following form:

$$\left.\frac{\partial C_{ijk}(x,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial C_{ijk}(x,t)}{\partial x}\right|_{x=L} = 0;\ C_{000}(x,0) = f_C(x);\ C_{ijk}(x,0) = 0.$$

Solutions of Eq. (16) could be written as

$$C_{000}(x,t) = \frac{F_{0C}}{L} + \frac{2}{L}\sum_{n=1}^{\infty} F_{nC} c_n(x) e_{nC}(t),$$

where

$$F_{nC} = \int_0^L f_C(v) c_n(v)\,dv,\ e_{nC}(t) = \exp(-\pi^2 n^2 D_{0L} t/L^2);$$

$$C_{i00}(x,t) = -2\pi\frac{D_{0C}}{L^2}\sum_{n=1}^{\infty} n F_{nC} c_n(x) e_{nC}(t)\int_0^t e_{nC}(-\tau) \times \int_0^L g_L(v,T) s_n(v)\frac{\partial C_{i-100}(v,\tau)}{\partial v}\,dv\,d\tau, \quad i \ge 1;$$

$$C_{010}(x,t) = -2\pi\frac{D_{0C}}{L^2}\sum_{n=1}^{\infty} n F_{nC} c_n(x) e_{nC}(t)\int_0^t e_{nC}(-\tau) \times \int_0^L s_n(v)\frac{C_{000}^{\gamma}(v,\tau)}{P^{\gamma}(v,T)}\frac{\partial C_{000}(v,\tau)}{\partial v}\,dv\,d\tau;$$

$$C_{020}(x,t) = -2\pi\frac{D_{0C}}{L^2}\sum_{n=1}^{\infty} n F_{nC} c_n(x) e_{nC}(t)\int_0^t e_{nC}(-\tau) \times \int_0^L s_n(v)\frac{C_{000}^{\gamma-1}(v,\tau)}{P^{\gamma}(v,T)} \times \left[C_{010}(v,\tau)\frac{\partial C_{000}(v,\tau)}{\partial v} + C_{000}(v,\tau)\frac{\partial C_{010}(v,\tau)}{\partial v}\right]dv\,d\tau;$$

$$\begin{aligned}C_{110}(x,t) = {} & -2\pi\frac{D_{0C}}{L^2}\sum_{n=1}^{\infty} n F_{nC} c_n(x) e_{nC}(t)\int_0^t e_{nC}(-\tau) \times \int_0^L s_n(v)\left[g_L(v,T)\frac{\partial C_{010}(v,\tau)}{\partial v}\right. \\ & + \frac{C_{000}^{\gamma}(v,\tau)}{P^{\gamma}(v,T)}\frac{\partial C_{100}(v,\tau)}{\partial v} + C_{100}(v,\tau)\frac{C_{000}^{\gamma-1}(v,\tau)}{P^{\gamma}(v,T)}\frac{\partial C_{000}(v,\tau)}{\partial v} \\ & \left. + \frac{C_{000}^{\gamma}(v,\tau)}{P^{\gamma}(v,T)}\frac{\partial C_{000}(v,\tau)}{\partial v} g_L(v,T)\right] dv\,d\tau;\end{aligned}$$

$$C_{000}(x,t) = \frac{F_{0C}}{L} + \frac{2}{L}\sum_{n=1}^{\infty} F_{nC}c_n(x)e_{nC}(t);$$

$$C_{001}(x,t) = L^{-1}F_{0C}\{1 - \exp[-q_C(x,T)I^*V^*V(x,t)]\};$$

$$C_{002}(x,t) = L^{-1}F_{0C}\{1 - \exp[-q_C(x,T)\,I^*V^*V\,(x,t)]\};$$

$$\ldots \quad \ldots \quad \ldots$$

Analysis of spatiotemporal distribution of dopant concentration has been done analytically using the second-order approximation of dopant concentration. Further the distribution has been amended numerically.

Discussion

In this paragraph, we analyzed redistribution of dopant in considered SH (see Fig. 1). Let us consider that the epitaxial layers have been processed by radiation before implantation of dopant. Further a dopant has been implanted in the EL_1. After this implantation microwave annealing of radiation defects in EL_2 has been done to decrease their quantity. It is known (see, for example, Sobolev 2010; Kozlivsky 2003) that radiation processing leads to acceleration of dopant diffusion. In this situation decreasing of quantity of radiation defects in EL_2 leads to decreasing of dopant diffusion coefficient in this layer in comparison with dopant diffusion coefficient before annealing. After this annealing of radiation defects in EL_2 let us consider microwave annealing of radiation defects in both epitaxial layers. During the second annealing, the quantity of radiation defects in the layers decreases and dopant distribution spreads. During processing of the SH and implantation of dopant it should be taken into account that thickness of EL_1 and energy of ions are not independent from each other. During annealing of radiation defects in both epitaxial layers dopant should achieve or almost achieves both interfaces (between both epitaxial layers and between EL_1 and *S*). If the dopant did not achieve the interfaces, it is practicably to use additional annealing of dopant to obtain the achievement. After the achievement, difference of properties between EL_1 and *S* leads to increasing of sharpness of *p–n* junction between the layers and homogeneity of dopant distribution in enriched area (Pankratov 2008a, b; Shalimova 1985; Pankratov 2005). The same situation one can obtain with another *p–n*

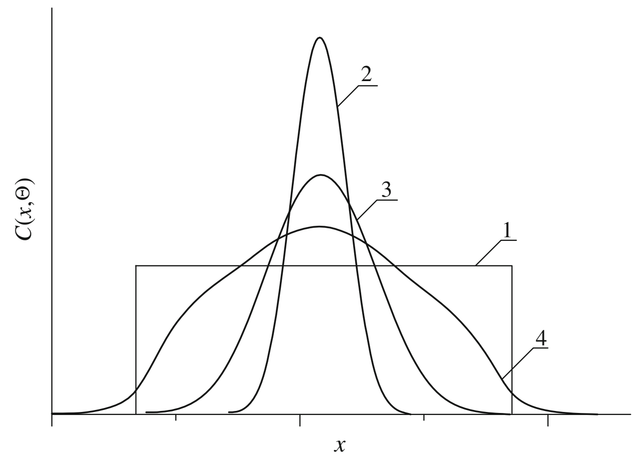

Fig. 3 *Curve 1* is idealized step-wise approximation of dopant concentration. *Curves 2–4* are real distributions of dopant concentrations for different values of times. The values increase with increasing of the numbers of curves

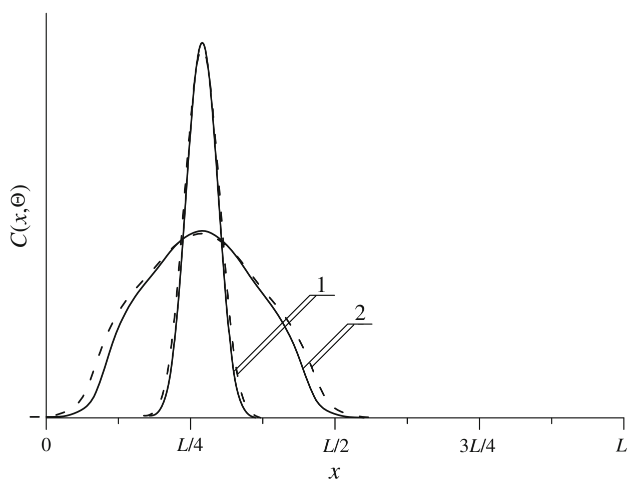

Fig. 2 *Curve 1* is implanted dopant distributions in homogenous material. *Curve 2* is implanted dopant distributions in the heterostructure in Fig. 3. *Solid lines* correspond to nonprocessed heterostructure. *Dashed lines* correspond to heterostructure after radiation processing of *EL*

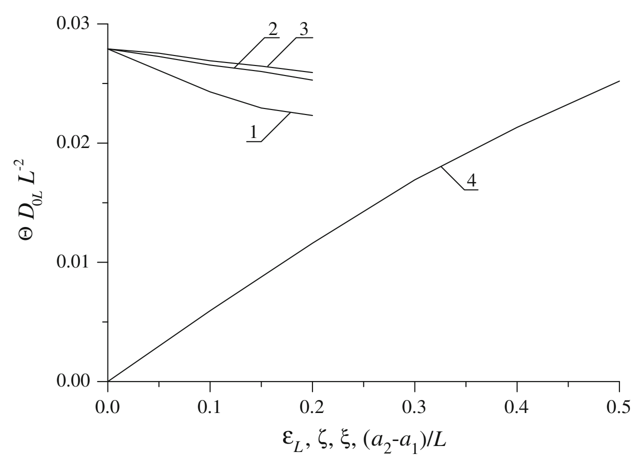

Fig. 4 Dependence of normalized compromised annealing time $\vartheta = \Theta D_0/L^2$, which was obtained by minimization of mean squared error Eq. (17), on several parameters. *Curves 1–3* are dependences of ϑ on ε_L, ζ and ξ for zero values of another parameters and relation $(a_2 - a_1)/L = 1/2$. *Curve 4* is dependence of ϑ on *a/L* for $\varepsilon = \zeta = \xi = 0$

junction (between EL_1 and EL_2) (Pankratov 2008a, b; Shalimova 1985; Pankratov 2005). In this situation, we obtain a heterobipolar transistor with higher sharpness of *p*–*n* junctions in comparison with *p*–*n* junctions in homogenous sample. At one time homogeneity of dopant distribution in enriched area increases. In our case the enriched areas for both *p*–*n* junctions coincide with each other. Radiation processing of doped area leads to increasing of the difference between diffusion coefficients of dopant in dopant-enriched area and nearest areas of SH. The increasing leads to increasing of the both above effects (increasing of homogeneity of dopant distribution in enriched area and sharpness of *p*–*n* junctions). Inhomogeneity of temperature distribution during microwave annealing also leads to increase in the difference between diffusion coefficients of dopant in dopant-enriched area and nearest areas of SH. However, the additional increasing could be obtained only for *p*–*n* junction, which was formed between EL_1 and *S*, because radiation defects also annealed in EL_2. Thereat additional difference between dopant diffusion coefficient in EL_1 and EL_2 due to microwave annealing is absent. Spatial distributions of dopant in considered SH (see Fig. 1) after preimplantation radiation processing of epitaxial layers and microwave annealing of radiation defects in the layers after the implantation are presented in Fig. 2.

Further let us optimize continuance of the additional annealing of dopant to shift *p*–*n* junctions to the interfaces between layers of the SH, for instance, when the dopant did not achieve the interface after annealing radiation defects. If annealing time is small, we obtain too inhomogenous distribution of dopant (see Fig. 3). If annealing time is large, we obtain too homogenous distribution of dopant (see Fig. 3).

Compromise annealing time has been obtained by minimization of the mean squared error between real spatiotemporal distribution of dopant concentration $C(x, t)$ and step-wise approximation of the concentration $\Psi(x)$ (Pankratov 2008a, b; Shalimova 1985; Pankratov 2005) (see Fig. 3)

$$U = \frac{1}{L}\int_0^L [C(x,t) - \psi(x)]^2 \, dx. \tag{17}$$

The results of minimization of functional Eq. (17) is presented in Fig. 4 as function of several parameters.

Conclusion

In this paper, we consider an approach to increase the compactness and homogeneity in dopant-enriched area of distribution fabricated by implantation bipolar transistor in semiconductor heterostructure using preimplantation radiation processing and microwave annealing.

Acknowledgments This work has been supported by grant of President of Russia (project № MK-548.2010.2).

References

Ahlgren T, Likonen J, Slotte J, Räisänen J (1997) Concentration dependent and independent Si diffusion in ion-implanted GaAs. Phys Rev B 56(8):4597–4603

Fahey PM, Griffin PB, Plummer JD (1989) Point defects and dopant diffusion in silicon. Rev Mod Phys 61:289

Gotra ZY (1991) Technology of microelectronic devices. Radio and communication, Moscow (in Russian)

Grebene AB (1983) Bipolar and MOS analogous integrated circuit design. Wiley, New York, p 894

Kozlivsky VV (2003) Modification of semiconductors by proton beams. Nauka, Sant-Peterburg (in Russian)

Lachin VI, Savelov NS (2001) Electronics. Phoenix, Rostov-na-Donu (in Russian)

Pankratov EL (2005) Influence of the spatial, temporal, and concentrational dependence of the diffusion coefficient on dopant dynamics: optimization of annealing time. Phys Rev B 72(7): 075201

Pankratov EL (2008a) Redistribution of dopant during microwave annealing of a multiplayer structure for production *p*-*n*-junction. J Appl Phys 103(6):064320

Pankratov EL (2008b) Redistribution of dopant, implanted in a multilayer structure for production of a *p*-*n*-junction, during annealing radiative defects. Phys Lett A 372(11):1897

Pankratov EL, Bulaeva EA (2012) Decreasing of quantity of radiation defects in an implanted-junction rectifier in a semiconductor heterostructure. Int J Micro-Nano Scale Transport 2(1):85

Pankratov EL, Spagnolo B (2005) Optimization of impurity profile for *p*-*n*-junction in heterostructures. Eur Phys J B 46(1):15

Pokotilo YuM, Petukh AN, Litvinov VV (2006) New hydrogen donors in silicon. Tech Phys Lett 29(19):26–30

Ryssel H, Ruge I (1978) Ion implantation. B.G. Teubner, Stuttgart

Shalimova KV (1985) Physics of semiconductors. Energoatomizdat, Moscow (in Russian)

Sisianu ST, Sisianu TS, Railean SK (2002) Shallow *p*-*n*-junctions in *Si*, manufactured by pulse photon annealing. Semiconductors 36(6): 581

Sobolev NA (2010) Engineering of defects in implantation technology of light-emitting structure with dislocation luminescence. Semiconductors 44(1):3

Volokobinskaya NI, Komarov IN, Matyukhina TV, Reshetnikov VI, Rush AA, Falina IV, Yastrebov AS (2001) Analysis of technological processses of manufacturing of high-power bipolar transistors. Semiconductors 35(8):1013

Zorin EI, Pavlov PV, Tetelbaum DI (1975) Ion doping of semiconductors. Energiya, Moscow (in Russian)